II. THEORY

We first pose the problem of shock temperature and formulate a theoretical basis for its solution. Let e, s, U, and u denote specific energy, specific entropy, shock velocity, and particle velocity, and let subscript o denote the constant state of stationary fluid in front of the shock. Then the Rankine-Hugoniot jump[6] conditions relating shocked and unshocked states,

$$vU = v_o(U - u) \tag{1}$$

$$uU = v_o(p - p_o) \tag{2}$$

$$puv_o = U(e - e_o + \tfrac{1}{2}u^2) \tag{3}$$

express the balance of mass, momentum, and energy across the shock discontinuity, and the inequality

$$s(e,v) > s(e_o,v_o)$$

expresses the second law of thermodynamics for the irreversible shock process.

Eliminating U and u from Eq. 3 gives the Hugoniot equation[7]

$$e - e_o = \tfrac{1}{2}(p + p_o)(v_o - v). \tag{4}$$

If an (e-p-v) equation of state satisfies the condition $(\partial^2 p/\partial v^2)_s > 0$, then Eq. 4 with $v < v_o$ defines the locus of compressed states on the (e-p-v) surface that can be reached from an initial condition (e_o,p_o,v_o) by single shocks. The (e-p-v) equation of state and Eq. 4 define this locus of shocked states as a curve in the (p,v) plane, $p = p_H(p_o,v_o,v)$, which passes through the point (p_o,v_o) and is called the Hugoniot curve centered at (p_o,v_o). The elimination of u from Eqs. 1 and 2 gives the equation of the Rayleigh line,

$$p - p_o = (U/v_o)^2 (v_o - v). \tag{5}$$

Since a shocked state satisfies Eqs. 4 and 5, the intersection of the Hugoniot curve centered on (p_o,v_o) and the Rayleigh line of slope $-(U/v_o)^2$ passing through (p_o,v_o) defines the mechanical thermodynamic state (p,v) behind a shock propagating at constant velocity U into a stationary state (p_o,v_o).

With the assumption of thermodynamic equilibrium behind a shock, the state variables of a nonreacting shocked fluid satisfy the following thermodynamic identities:

$$de = Tds - pdv \tag{6}$$

$$T(s,v) = \left(\frac{\partial e}{\partial s}\right)_v \tag{7}$$

$$p(s,v) = -\left(\frac{\partial e}{\partial v}\right)_s \tag{8}$$

For thermomechanical processes, a knowledge of e, s, T, p, and v provides a complete characterization of a thermodynamic state. Thus, the (e-s-v) equation of state is called complete because of the identities 7 and 8 that define the (T-s-v) and (p-s-v) equations of state, but all other equations of state among these variables are incomplete. The (e-p-v) equation of state is incomplete because it cannot be used to calculate temperature and entropy without additional data. Similarly, the (T-p-v) equation of state is incomplete because it cannot be used to calculate energy and entropy without additional data. However, a knowledge of any two incomplete equations of state provides a complete characterization because of the identities of thermodynamics.

The objective of the present work is to use shock wave and low pressure data to characterize completely the high pressure environment in the kilobar regime without additional thermodynamic assumptions. Since shock temperature cannot be measured directly with present-day techniques and cannot be calculated from knowledge of the energy along a Hugoniot curve, it is necessary to construct the (T-p-v) equation of state. Such a construction must be based on the mechanical properties of shocked states. At present the only feasible way to achieve this objective is to construct the (e-p-v) equation of state first, and then use it with the identities of thermodynamics to calculate the (T-p-v) relationship. Hugoniot curves form the basis of the experimental method of constructing the (e-p-v) equation of state using shock wave data; the relationship between the (T-p-v) and (e-p-v) equations of state forms the basis for calculating the temperature of shocked states.